Fourier Series/Gibbs phenomenon

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Background info…

- First appeared in the mid 18th century when Euler observed that a linear function can be written as an infinite summation of waves.
- Is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical

- They’re is used in the analysis of signals in electronics.
- It uses a global method, meaning that the function gets its information from the whole domain not just one point.

About fourier series

- The Fourier series are a set of infinite trigonometric series.

sin(pi x)+1/3sin(3pi x)+1/5sin(5pi x)+1/7sin(7pi x)+……..+ (Infinity)

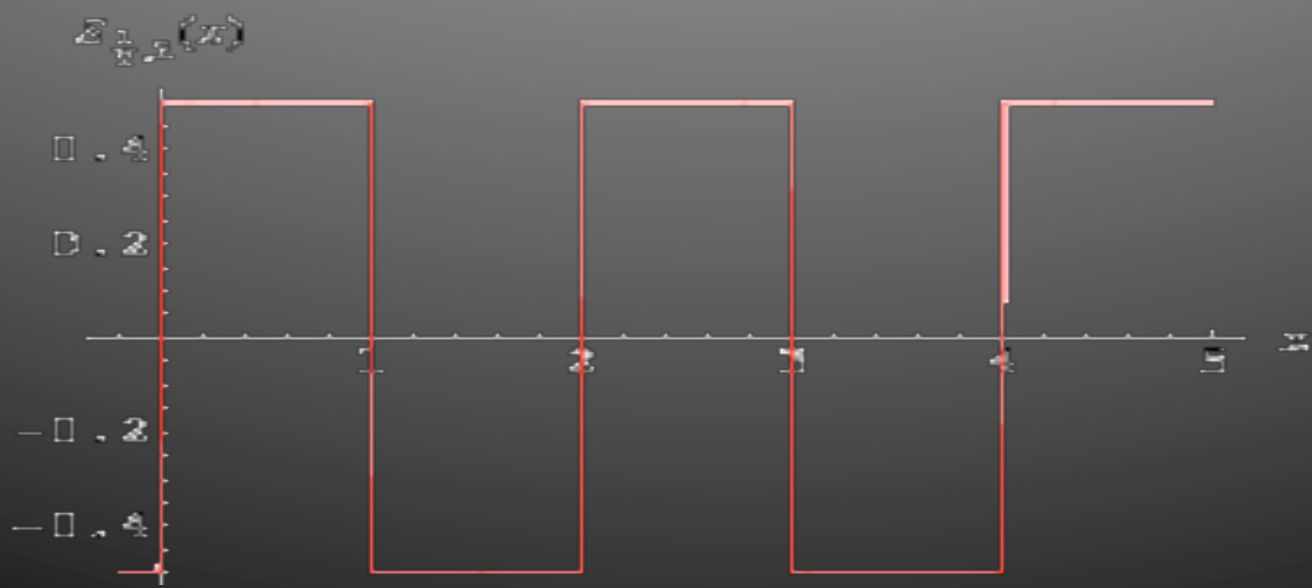
As it takes on more and more terms, the combined signal becomes to look like a square wave function

What this means

- As more terms are added to the series, the graph starts gaining the shape of what we would call a square wave function

What is a square wave function

- A square wave function is a periodic waveform consisting of instantaneous transitions between two levels



Consider our function

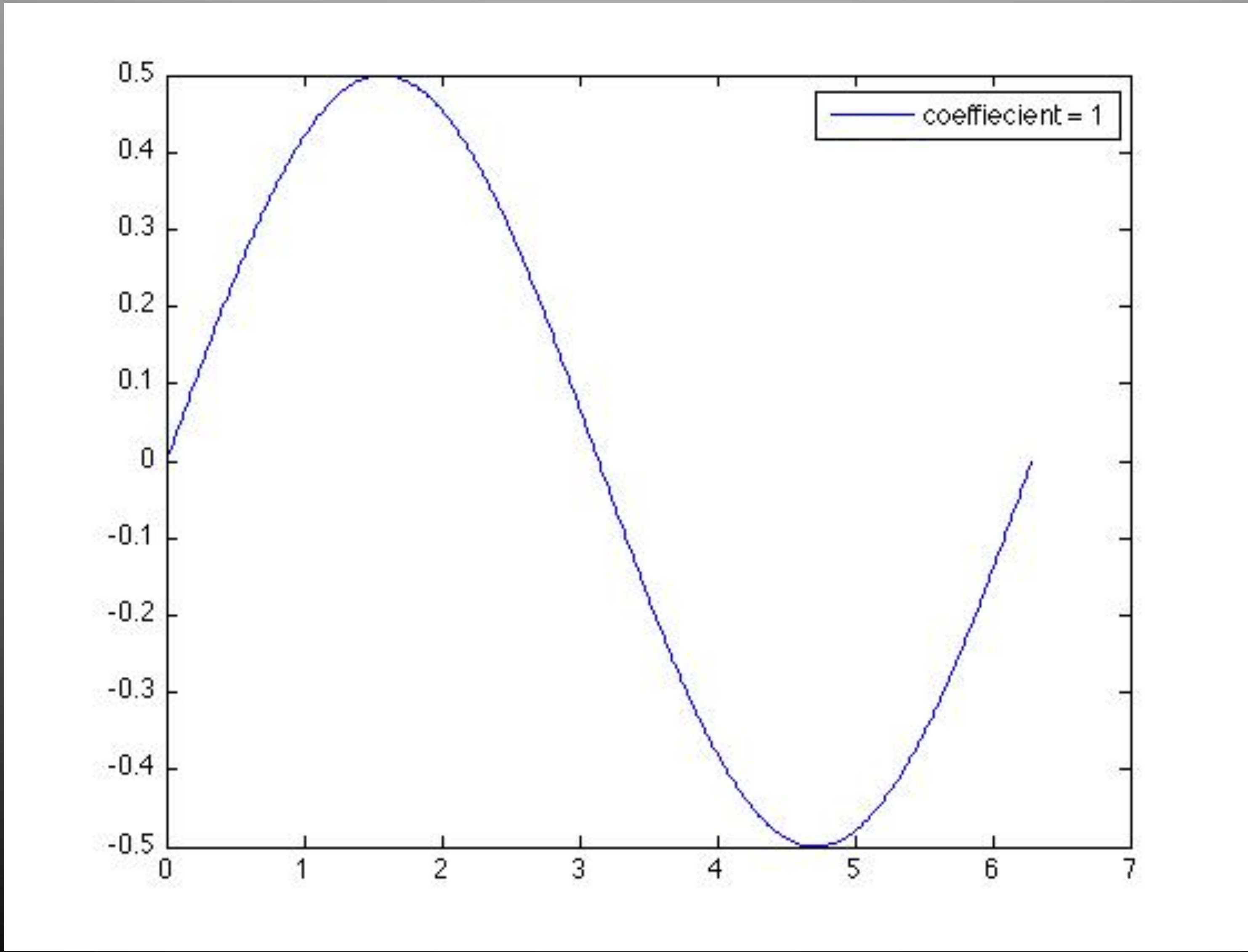
sin(pi x)+1/3sin(3pi x)+1/5sin(5pi x)+1/7sin(7pi x)+……..+ (Infinity)

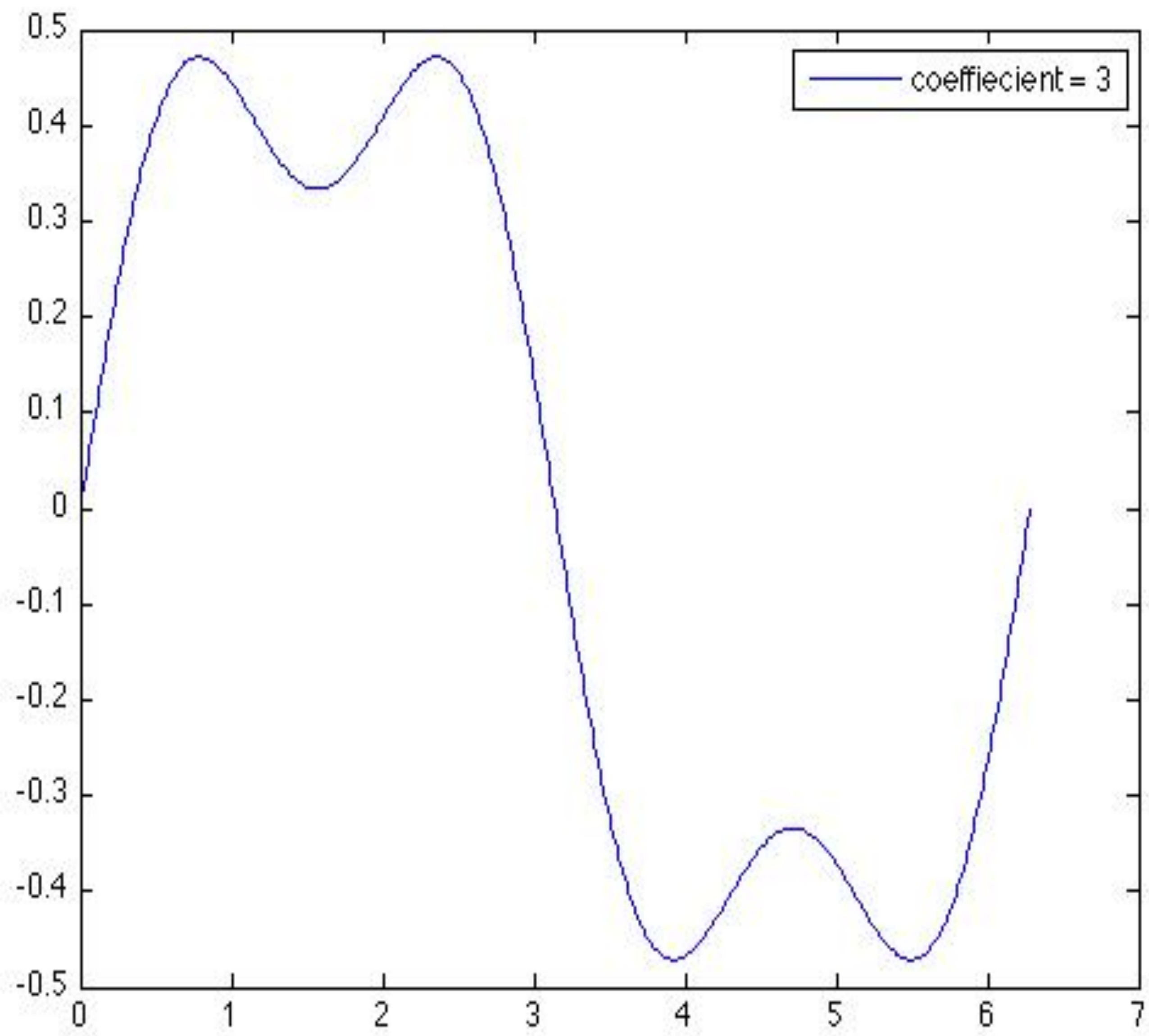
Matlab code

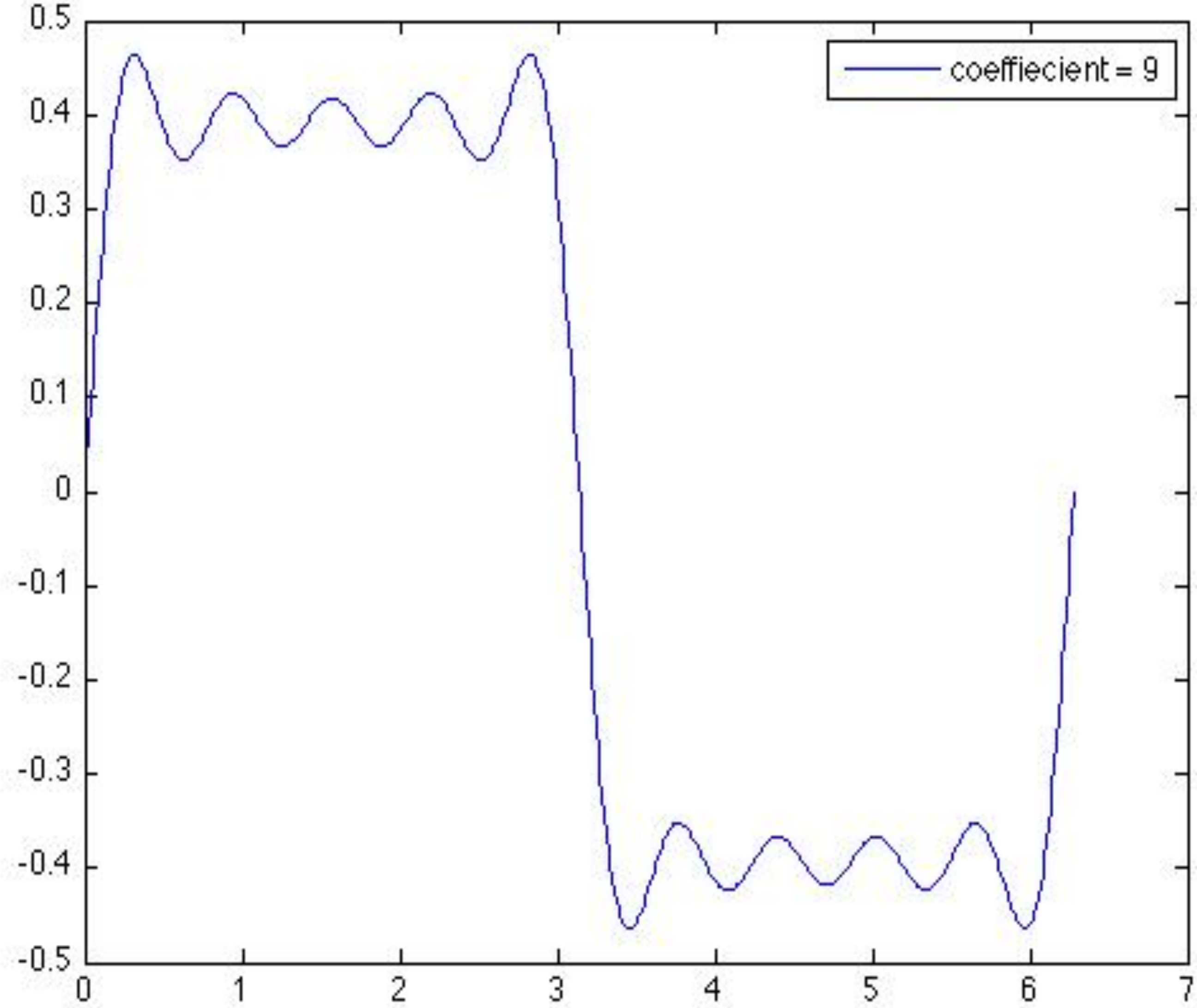
- J= 500 %number of points
- x= linspace(0, 2*pi, J);
- f= sign(x); %returns array same size as x
- kp=0.*x; %multiplies everything by x starting with 0
- t= 150
- for k=1:2:t
- kp=kp+(1/2)*sin(k*x)/k;
- end

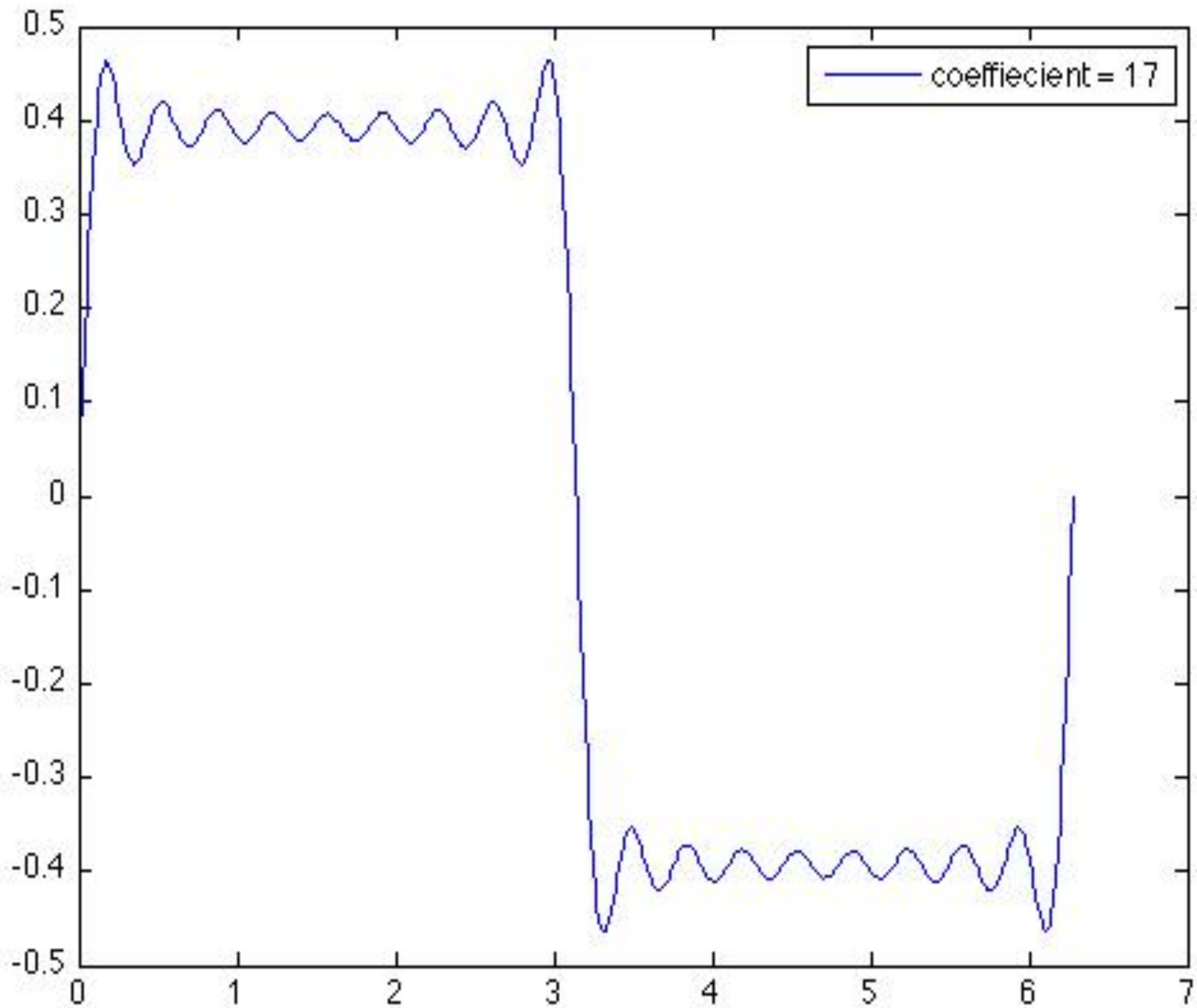
- plot(x, kp)

Let’s take a closer look

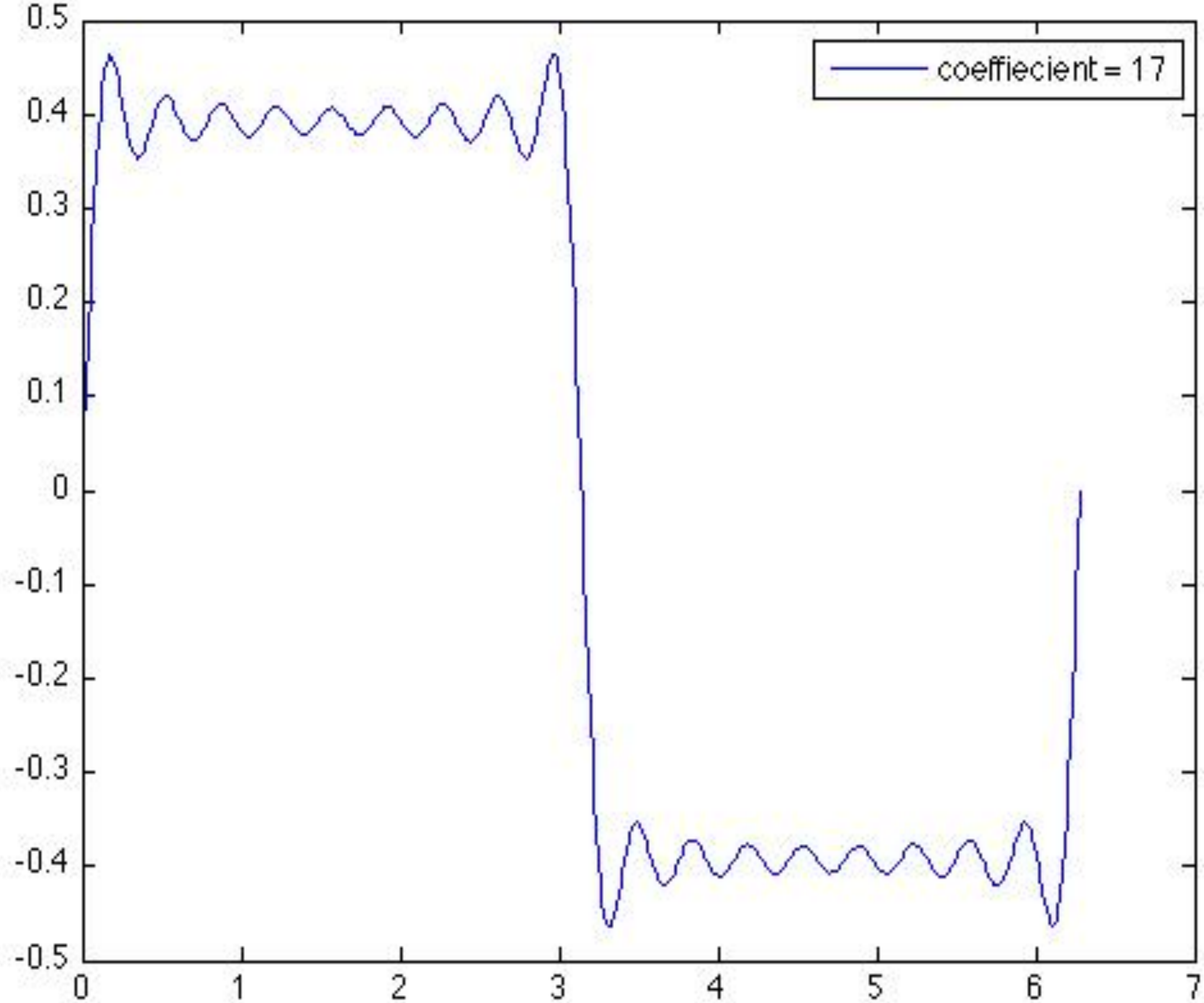



coeffiecient = 3
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7

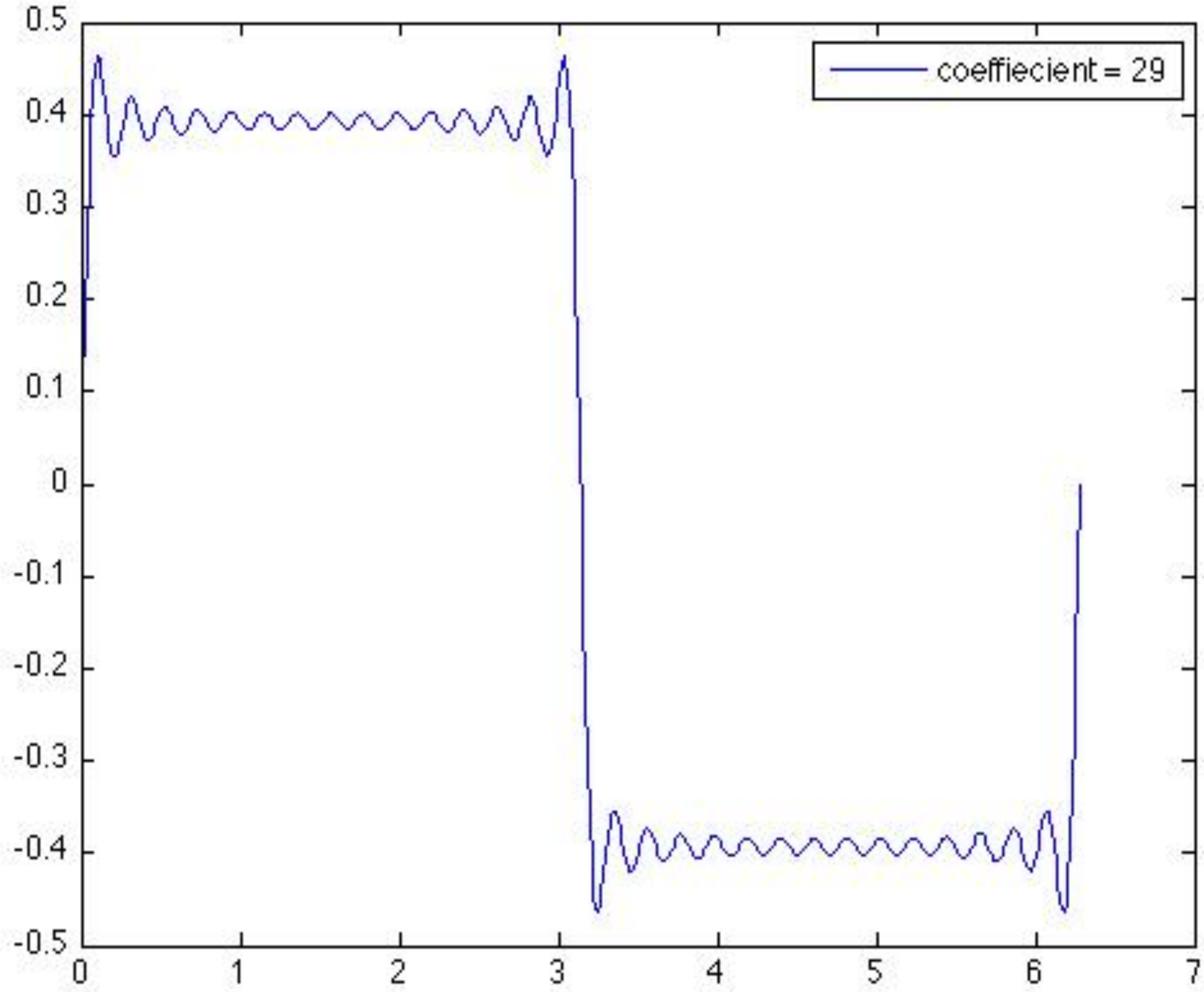

coeffiecient = 9
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7



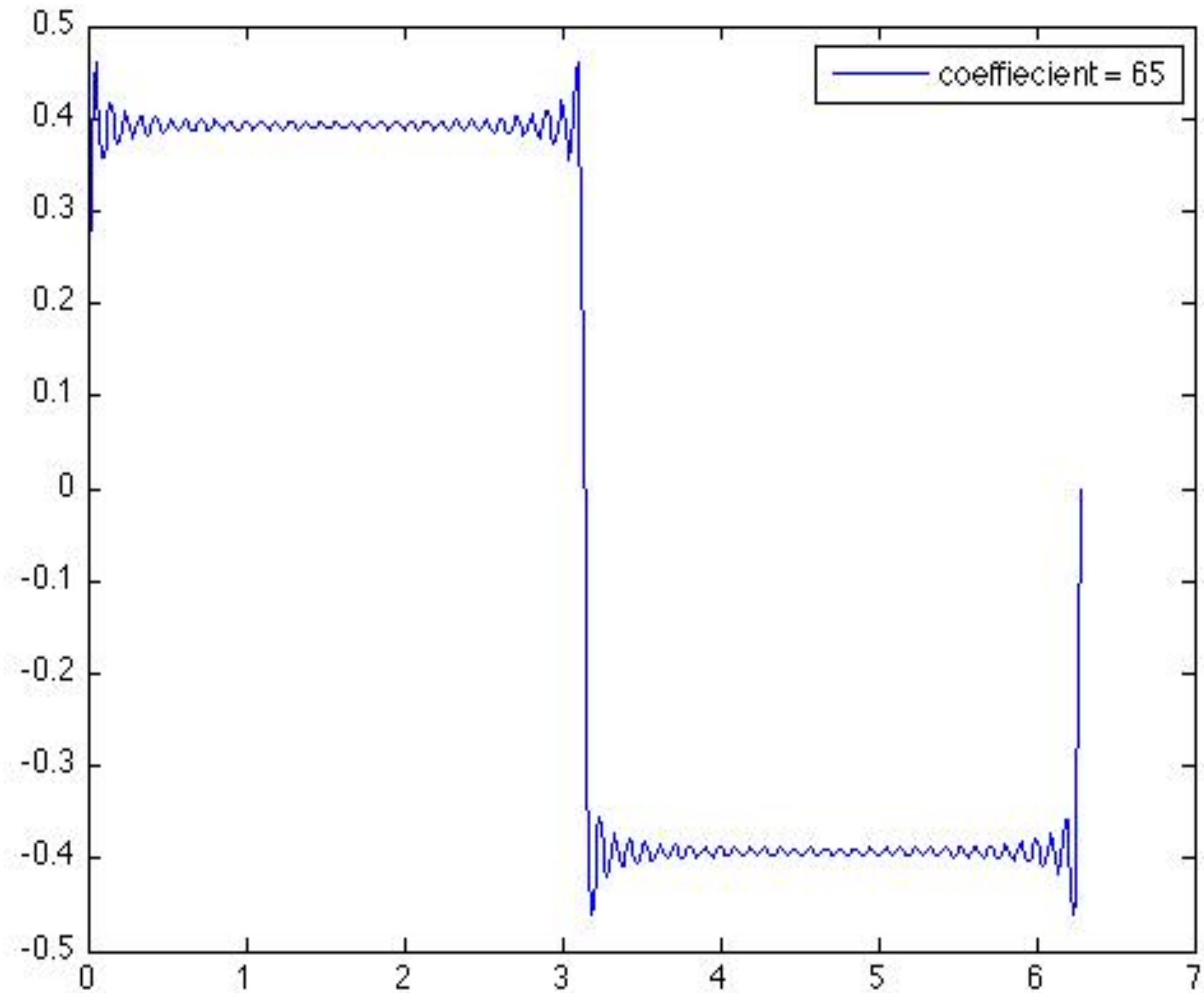
coeffiecient = 17
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7

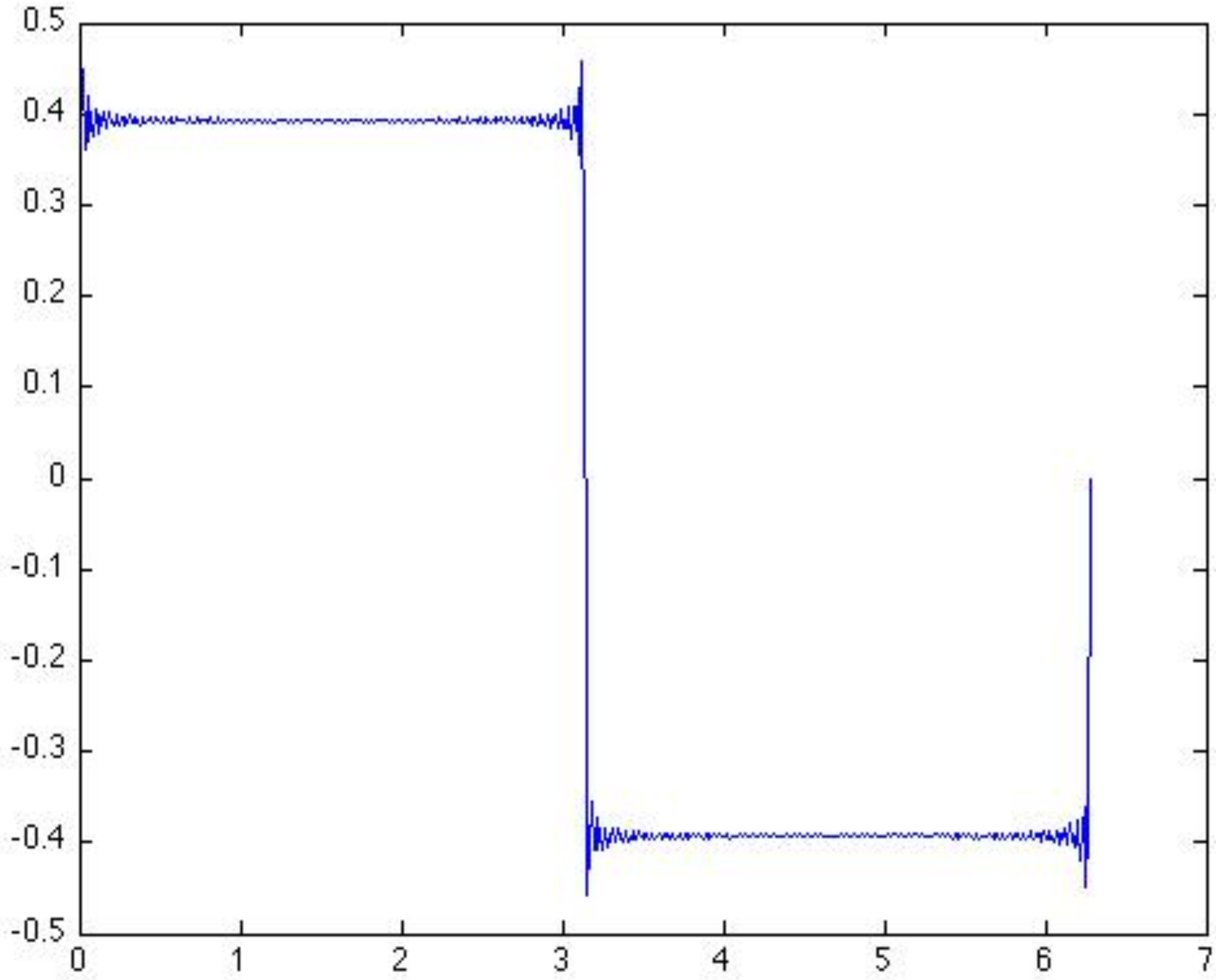


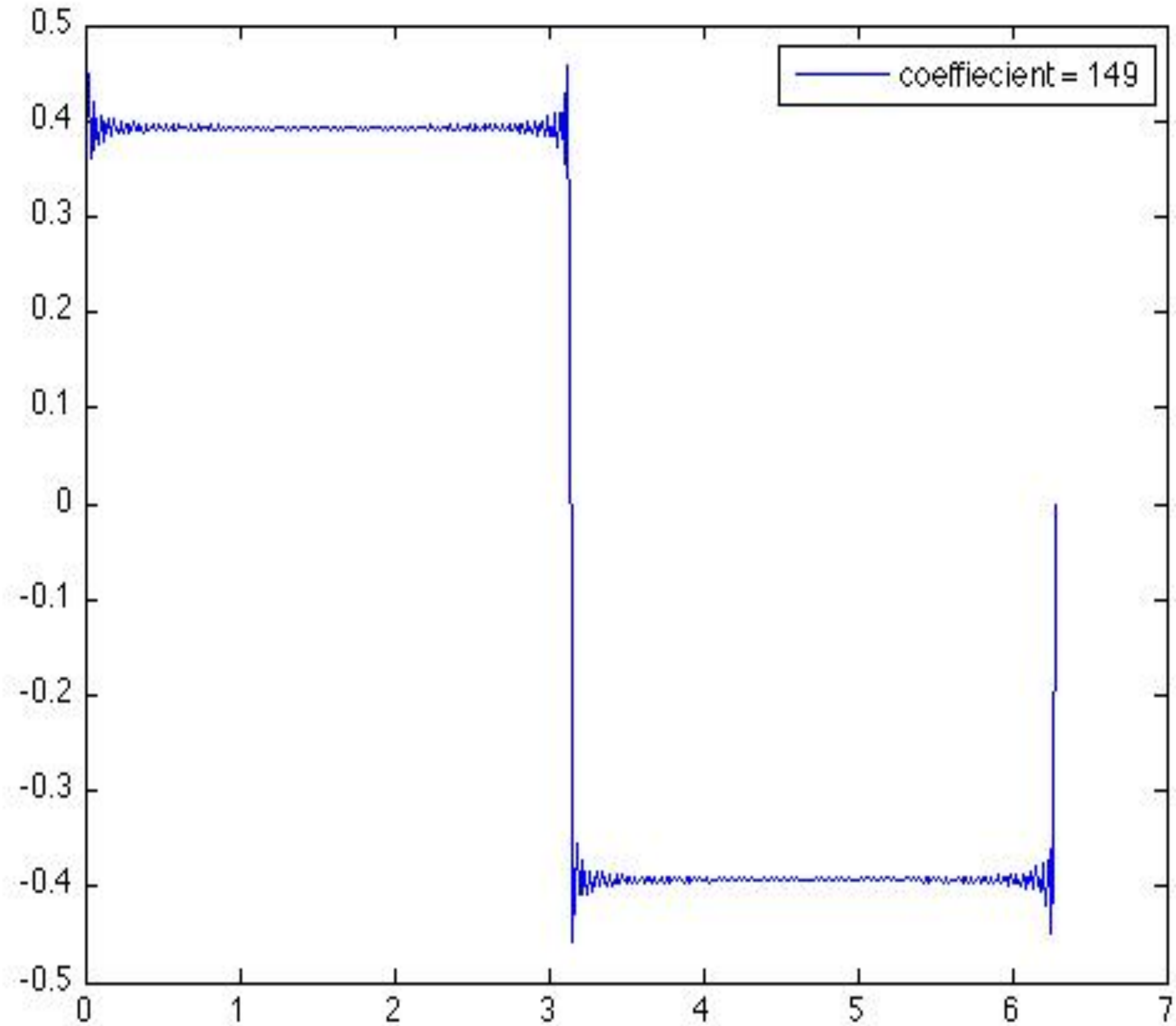
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7
coeffiecient = 17



coeffiecient = 29
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7


coeffiecient = 65
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7



coeffiecient = 149
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
1
2
3
4
5
6
7

What's happening

- As it takes more and more terms, oscillation does not go away, it slides closer to the discontinuity. This is what creates what we call the Gibbs phenomenon.

Continuing project

- In order to solve the Gibbs phenomenon, we need to pay close attention to the errors. This leads to the Gegenbauer postprocessing.