The Effect of Interactive Applets in Mathematics Teaching

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Relationship to conference goals

This paper presents a discussion of the author’s recent and upcoming research on the effect of web-based interactive applets on mathematics pedagogy. The paper relates to the conference topics of Application Domain (K-12) and Technology (Multimedia, Simulations, Visualizations, Web Technologies).

Introduction

Interactive applets, which are typically web-based, are the most recent manifestation of technological tools used in mathematics education. They are unique in their focus on small interactive units, due in large part to the current nature of web-access where large programs are unwieldy and quickly exhaust download capabilities and user patience. Well designed interactive applets enable students to engage in investigations of mathematical relationships without having to spend a lot of time learning how to use the tool that creates the various representations of these relationships. For example, the volume and surface area activity (NCTM, 2001), shown in Figure 1, enables students to investigate multiple representations of volume and surface area relationships. The focused and user-friendly nature of well-designed interactive applets may make them appropriate as models of mathematical concepts and of new pedagogical directions, as is the case at the Illuminations.nctm.org web site of NCTM (2001), where interactive applets such as the one in Figure 1 are used to demonstrate the pedagogical directions outlined in the Principles and Standards for School Mathematics document (NCTM 2000).

Related research

Some research indicates that “our thinking is deeply molded by material devices and socio-technical collectives” (Levy, 1993, p. 10). Perhaps well designed and easily accessible interactive applets that allow students and teachers to use and explore mathematical relationships may affect what Levy refers to as the “cognitive ecology” of learning environments (Levy, 1993). It may be possible that “new forms of representation change the mathematics being taught” (Confrey, 1993, p. 335). New media not only enable us to express our ideas in new ways – they also affect the ideas we have. “We don’t always have ideas and then express them in the medium. We have ideas with the medium. Making progress is an episode of materially mediated thinking – reasoning or coming up with a new idea – happens jointly in the mind and in the medium at every stage” (DiSessa, 2000, p. 116). And perhaps more importantly, some research indicates that there may be a spillover effect, where learning to think with material devices affects our thinking even when the devices are not present (Borba & Villarreal, 1998). However, some research also
indicates that attempts to change teachers’ mathematics conceptions and teaching practice “are minimally effective, in part because teachers filter what they learn through their existing beliefs” (Stipek et al 2001, 214) and teachers assimilate new ideas without substantially altering existing beliefs that drive their practice (Cohen & Ball 1990).

This paper briefly reports on recent case studies of three grades 5-6 and one grade 10 teacher using web-based interactive applets in their teaching of mathematics. The paper also describes an upcoming study of the affect of the availability of interactive applets on the pedagogical thinking of teachers. In the cases described below, teachers were first introduced to a variety of web-based interactive applets. The teachers selected which applets to use in their teaching based on the mathematics topics in their next unit of study. The three grades 5-6 teachers chose an applet dealing with measurement (Figure 2)(ExploreLearning, 2000a). The grade 10 teacher chose an applet dealing with the vertex form of a quadratic function. (Figure 3) (ExploreLearning, 2000b).

Brief summary of the effect of interactive applets in grades 5-6 mathematics classrooms

The grades 5-6 teachers used the Maximize Area applet (Figure 2) in the context of teaching a unit on measurement. The teachers met and decided on the general direction for the classroom and online activities. The grade 5 teacher then used these ideas to develop a detailed lesson that was shared with the grade 6 teachers. The lesson plan started with the following problem: “We’re going to build a pen for your dog. Your parents are going to go out and buy 24 metres of fence. You’re going to build it with four sides. How are you going to get the biggest area for your dog to play?”

In the grade 5 classroom, the teacher drew one possible pen configuration on the board and then asked the students to find and draw all possible configurations on grid paper. Then students developed a table of values, recording the dimensions of the pens they had drawn, and they plotted area versus length on a graph. Students had previous experiences plotting straight line graphs and were surprised that the area graph was curved. “And they said oh that’s kind of a neat shape because they’re used to […] getting a straight line […] and the reason I wanted to do that [in class] was because once they go to the computer that is what they are going to see. […] I didn’t want them to go to the computer right away because […]

![Figure 2. Maximize Area applet.](image)

![Figure 3. Quadratics: Vertex Form applet.](image)
I think they would see the graphs and say that’s neat but not relate it to any meaning […] I think that was a big key for their understanding.”

The next day, the grade 5 students went to the computer lab, and they were quickly introduced to the features of the Maximize Area web-based applet, with the teacher drawing parallels with the work that students had done in class in drawing diagrams, finding values, and creating graphs. In the computer lab, students were asked to solve the problem where the perimeter is 20 metres. The teacher was impressed with how well students understood and solved the problem of finding the pen with maximum area. “They liked seeing this, they liked seeing the area changing.” The teacher was also surprised “because I have some grade fives that are not top grade fives but all of them got it.” Some students asked, “is the biggest area always a square? […] Wouldn’t a circle be bigger? […] They were really thinking.”

In contrast, the grade 6 teachers decided to do the computer lab component of the lesson before they did the classroom component. They did this because of minimal access to the computer lab and because of the tight timelines of other classroom activities. In the computer lab, the grade 6 teachers noticed that some students were not focusing on the problem at hand when they were using the Maximize Area activity. “Some of them thought it was a toy […] changing sliders all over the place.” The grade 6 teachers stated that they felt students going to the computer lab were expecting something different, “it seems they were expecting a game or a fun activity.” They noted that the activity worked better with the grade 5 students because “your kids knew what to expect when they went to the lab because they had done those kind of activities (in the classroom), and they knew what the problem was all about.” The grade 6 teachers said that next time they would reverse the order of the lesson activities, to match the sequence used by the grade 5 teacher. The subsequent classroom component of the lesson worked well with the grade 6 students and the grade 6 teachers felt that students learned a lot about the relationship between area and perimeter.

The Maximize Area applet may have acted as a pedagogical model for teachers’ classroom practice as the approach to teaching about area and perimeter described above varies from the norm. In previous years, teachers did not focus on the relationships between area and perimeter. Their typical approach to area and perimeter is more reliant on the development and use of formulas, with minimal emphasis on the relationship between area and perimeter. The grade 5 teacher said, “I think I do more the traditional way, separating the two.” One of the grade 6 teachers said, “I do perimeter first.” The grade 5 teacher reflected on the effect of this difference on students: “When I introduced area and perimeter [in previous years] they didn’t see a relationship between the two, but when you fix one at a given constant, and you can manipulate the other, they really found that pretty neat, especially when you apply it to something outside of math [like the dog pen problem].”

**Brief summary of the effect of interactive applets in grade 10 mathematics classrooms**

In the case of the grade 10 teacher, the effect of using the Quadratics: Vertex Form applet (Figure 3) appears to have shifted the pedagogical focus from studying the individual and isolated effects of coefficients of quadratic equations to a holistic and dynamic exploration of relationships between quadratic equations and graphs. The teacher used the applet in a lab setting where each student worked on their own computer. Students however collaborated in some cases to discuss what they discovered and to check their answers and understanding. Students were provided with a set of worksheets prepared by the teacher. The worksheets included:

- An explanation of how to log in to the school network and how to access the applet online
A set of guided investigations, questions and instructions

A self-test section where students manually graphed quadratic functions in vertex form on a grid and then used the applet to check their answers

As students worked through the worksheets, some individually and some in pairs, the teacher was interested in seeing whether students would be able to follow the worksheet instructions and complete the tasks successfully with minimal teacher intervention. The teacher seemed surprised at how focused the students were and how well they seemed to understand. However, he indicated that he would withhold judgement until he saw how well students would do on an upcoming unit test.

The teacher used the same unit test that he had used in a previous semester with a different class and compared the results of the two classes. The teacher noted that the test scores on the topic of the vertex form of a quadratic of his current students were significantly higher than the test results of students in the previous semester who used graphing calculators rather than the Quadratics: Vertex Form applet (approximately a 30% difference in test scores on average). Although student ability can vary from class to class, the teacher felt that the two classes were generally of similar ability.

In previous semesters, the teacher taught the vertex form of a parabola using graphing calculators where over a sequence of three different classroom periods, students explored the effect of coefficients a, h and k, respectively, using the graphing calculator and then moving to paper and pencil graphs. A fourth classroom period was used to consider the effect of all coefficients simultaneously. When using the Quadratics: Vertex Form applet, the teacher spent only two classroom periods on the topic: one in the lab (as described above) and one in the classroom to reinforce concepts using paper and pencil activities and graphs. The teacher noted that his approach changed when using the Quadratics: Vertex Form applet in that he started with the big picture first (exploration of relationships between all there coefficients and the graph) rather than considering each coefficient in isolation. He also noted that he spent much less time teaching this topic (about half the time). The teacher felt that in contrast to the Quadratics: Vertex Form applet, his previous use of graphing calculators did not allow students to see the dynamic nature of relationships between coefficients and graphs. Also, the graphing calculator user interface is relatively primitive.

Discussion

Both studies involved the use of a single web-based applet on a limited basis. The studies offer some support for existing research that shows that material resources affect the mathematics taught (Levy, 1993; Confrey, 1996; diSessa, 2000) by shifting to some degree the mathematics teaching focus from isolated concepts to relationships among concepts. However, the specific effects in each of the classrooms, including the level of pedagogical success experienced by the teachers, varied. More research is needed to explore the potential use of interactive applets as models of mathematics relationships and teaching practice.

Upcoming research

In 2002-2003 a follow-up study will be conducted to investigate the nature of the pedagogical thinking, and more specifically the lesson planning, of mathematics teachers when designing lessons on given topics, with and without access to the interactive applets shown in Figure 3 and Figure 4 (MathAdventures.com, 2002). Does the availability of interactive applets that may be used to explore
mathematical relationships affect the potential of student mathematical thinking and performance that is facilitated in the lessons designed by teachers?

The study will involve 16 teachers who will be presented, in sequence, with two of the four different lesson planning tasks described in Table 1.

<table>
<thead>
<tr>
<th>Task</th>
<th>Resources</th>
<th>Mathematics Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Any resources that the teacher decides to use.</td>
<td>The experimental and theoretical probability of the sum of two dice.</td>
</tr>
<tr>
<td>A2</td>
<td>The applet shown in Figure 4. Any other resources that the teacher decides to use.</td>
<td>Graphing the vertex form of a quadratic function, ( y = a(x-h)^2 + k ).</td>
</tr>
<tr>
<td>B1</td>
<td>Any resources that the teacher decides to use.</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>The applet shown in Figure 3. Any other resources that the teacher decides to use.</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1. Lesson Planning Tasks.**

The 16 teachers will be organized into four groups, based on the sequence of two lesson planning tasks they will complete, as shown in Table 2. With this grouping the study should be able to determine whether the order of the tasks has an effect and whether the availability of Interactive Applets in the first task has an effect on the second task.

The study has four stages. Structured, task-based interviews (Goldin 2000) will be used in the first stage of the study. Each of the teachers will be interviewed for 1 hour each, 30 minutes for each of the two tasks. A laptop will be available to demonstrate the interactive applets. Each of the 30 minute sections of the interviews will be structured as follows:

- The lesson planning task will be presented.
- Teachers may ask clarification questions.
- Teachers will be asked the following sequence of questions:
  - What do you know about the topic?
  - What are your expectations for students?
  - What introductory activity would you use to set the stage for the lesson?
  - What sequence of activities would you plan?
  - How will you know whether students have met your expectations?

In the second stage of the study, lesson plans described by teachers will be analyzed in terms of the level of student mathematical thinking and performance that they facilitate, based on the four-level scheme described below (Gadanidis 2002).

<table>
<thead>
<tr>
<th>Level</th>
<th>Question</th>
<th>Emphasis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>what?</td>
<td>Recalling mathematical facts and simple skills.</td>
</tr>
</tbody>
</table>

![Figure 4. Probability applet.](image)
<table>
<thead>
<tr>
<th></th>
<th>how?</th>
<th>Applying mathematical procedures to solve routine problems.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>why?</td>
<td>Understanding and explaining mathematical relationships.</td>
</tr>
<tr>
<td>4</td>
<td>what if?</td>
<td>Extending understanding to new contexts or more general cases.</td>
</tr>
</tbody>
</table>

Table 3. Levels of student thinking and performance.

In the third stage of the study, each teacher will be interviewed for an additional 30 minutes. The analysis of their lesson planning will be shared and questions will be asked to determine:

- The extent to which the analysis seems appropriate to the teachers.
- Other insights or explanations they may have that would enrich the analysis.

In the last stage of the study, a model will be developed to describe the effect of the availability of interactive applets on the lesson planning of mathematics teachers.

References


